Engineering Notes For Design With
Concrete Block Masonry

MASONRY CHRONICLES

Summer
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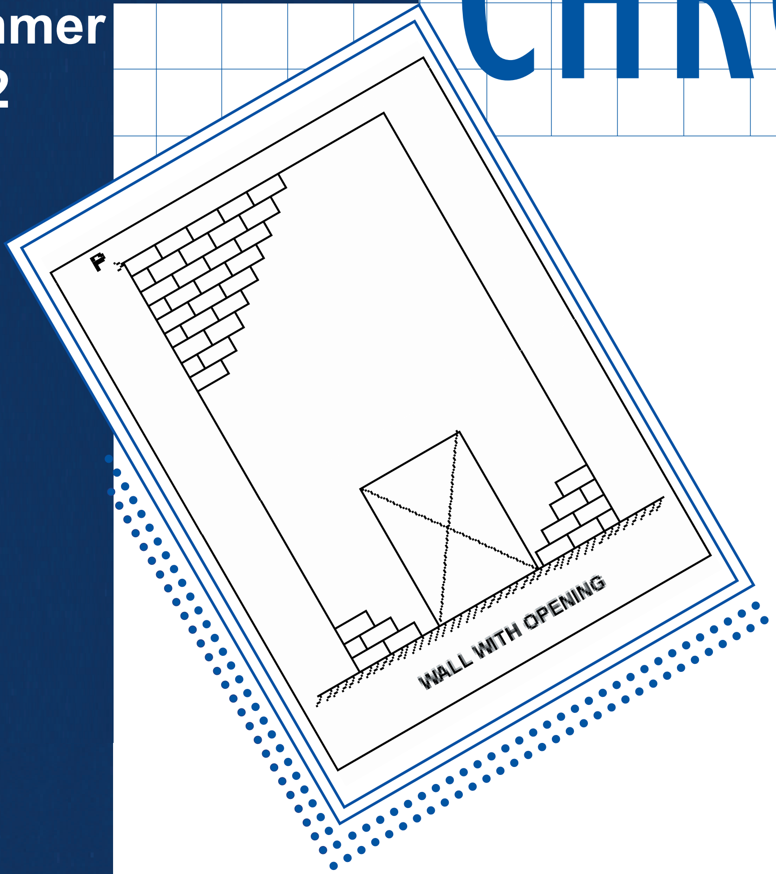

RIGIDITY CALCULATIONS FOR WALLS WITH OPENINGS

The *"Spring 2002 Masonry Chronicles"* issue addressed the design of piers and inherent in it was rigidity calculation for piers. This issue basically addressed the rigidity of shear walls in general, but not the design of shear walls.

Masonry walls often have openings in them for doors, windows or openings to accommodate equipment etc. Calculation of rigidities for these walls is complex and tedious. This article provides a background on the rigidity of walls, the components contributing to it and influence of openings on rigidity. Approximate method for calculating rigidity, which is useful for preliminary design, is also proposed.

INTRODUCTION

Walls, when subjected to in-plane lateral loads, undergo deflection. This deflection is a result of the wall behaving in flexural mode and shear mode. The prime behavioral mode is dependent on height to length ratios. Most walls neither behave in pure flexural mode, nor in pure shear mode. Their overall behavior is normally a combination of the two modes.

Wall rigidity is the amount of force required to deflect the wall by one unit.

Calculation of rigidity primarily serves two purposes :

1. distribution of lateral loads to various lateral load-resisting elements and 2. to calculate overall deflection (drift) of the system. While out-of-plane forces and behavior of the wall subjected to them may be important, it is not important in rigidity calculations for resisting in-plane forces.

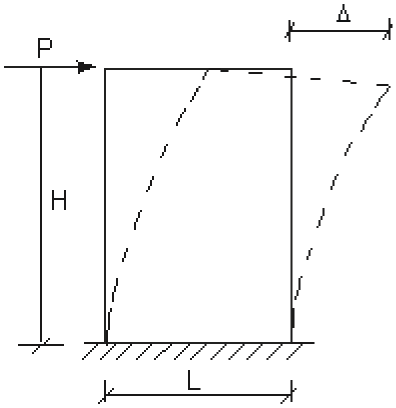

Figure 1: Shear Wall Deflection

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A cantilever shear wall subjected to lateral load will deflect "Δ". Considering flexural and shear deformations, Δ for a unit lateral load can be calculated by:

$$\Delta = \frac{H^3}{3E_m I} + \frac{1.2H}{AE_v}$$

A = t x L where $I = \frac{tL^3}{12}$ for uncracked section.

t = thickness of wall

E_v = shear modulus = 0.4 E_m

Substituting these values in the above equation:

$$\Delta = \left[\frac{4H^3}{E_m t L^3}\right] + \left[\frac{3H}{E_m t L}\right] = \frac{1}{E_m t}\left[4\left(\frac{H}{L}\right)^3 + 3\left(\frac{H}{L}\right)\right]$$

Flexural deformation Component

Shear Deformation Component

Similarly for "piers" or walls with top and bottom edges fixed against rotation, "Δ" is given by:

$$\Delta = \frac{1}{E_m t}\left[\left(\frac{H}{L}\right)^3 + 3\left(\frac{H}{L}\right)\right]$$

Whether *flexural deformation* governs or *shear deformation* governs, is dependent on the $\frac{H}{L}$ Ratio.

For a given wall, rigidity $= \frac{1}{\Delta}$ therefore, the lesser the deflection, the more rigid the wall. This can be intuitively seen by imagining trying to deflect a wall in its own plane. A higher $\frac{H}{L}$ ratio will require less force to deflect the wall than wall with a lower $\frac{H}{L}$ ratio by the same amount.

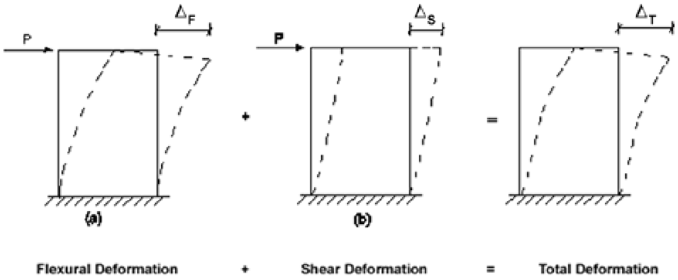

Figure 2: Modes of Deformation

Schematically, the modes of deformation are shown in Figure 2.

$$\Delta_F + \Delta_S = \Delta_T$$

As shown in Table 1, even for squat walls, i. e., $\frac{H}{L} \approx 1.0$, flexural deformation is 57% of the total. Whereas with a $\frac{H}{L}$ ratio of 2.5, which is not uncommon in 2-3 story tall buildings, flexural deformation is almost 90% of the total deformation. The purpose of Table 1 is to show the deformation multiplier, not to calculate the actual deformation. To calculate deformation, actual lateral load, E_m and t must be used. Table 1 is for cantilever walls. A similar table could be developed for other conditions.

Table 1: Deformation Components

			% of Total "Δ"	
$\frac{H}{L}$	$4\left(\frac{H}{L}\right)^3$	$3\left(\frac{H}{L}\right)$	Flexural	Shear
0.25	0.06	0.75	7	93
0.50	0.5	1.5	25	75
1.0	4.0	3.0	57	43
1.5	13.5	4.5	75	25
2.0	32.0	6.0	84	16
2.5	62.5	7.5	89	11
3.0	108	9.0	92	8

For Single story buildings with long walls, where length between control joints could be 1.5 times the height or $\frac{H}{L}$ ratio of 0.67, shear deformation predominates and flexural deformation concerns should be of less importance.

Although failure in shear is undesirable as it is considered non-ductile (brittle), the behavior of the wall due to its geometry cannot be changed. A well designed and detailed wall for anticipated shear demand should perform well.

Influence of Openings on Rigidity

As can be seen from the above discussion, shear deformation is a function of cross-sectional area of the wall for a given height. Therefore in squat walls, openings will reduce the cross-sectional area and the deflection will increase proportionally as opening sizes increase.

For flexure dominated walls, since the deformation is a function of moment of inertia, for a given height, the influence of openings on deflection is less if located centrally in the wall as the influence on reduction in moment of inertia is minor.

We will consider two cases of walls to demonstrate the effect of openings on the rigidity:

1. where $\frac{H}{L}$= 0.67

2. where $\frac{H}{L}$= 1.5

<u>**Case 1**</u>**:** Wall with $\frac{H}{L}$ = 0.67

Consider the following 8" CMU solidly grouted wall.

E_m = 1.5 ksi.

E_v = 0.6 ksi.

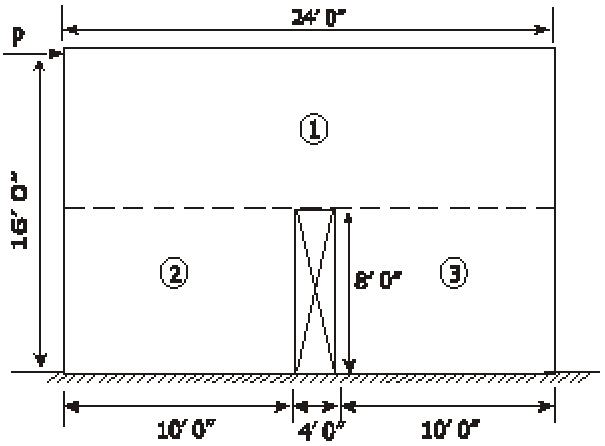

Figure 3: . Wall Elevation (Case 1)

$$\frac{H}{L} = \frac{16}{24} = 0.667$$

t = 7.625 inches

Substituting all the values, <u>for a 1K force</u>, for a solid wall,

$$\Delta_r = \frac{1}{1.5(7.625)}\left[4(0.667)^3 + 3(0.667)\right]$$

$$= \frac{1}{11.44}[1.19+2.00] = 0.28 \text{ in.}$$

Please note that 63% $\left(\frac{2.00}{3.19}\right)$of the deflection is due to shear.

$$\therefore R_{solid} = \frac{1}{\Delta_T} = \frac{1}{0.28} = 3.57$$

<u>Wall with Opening</u>

To account for the opening, the wall can be split into three portions ①, ②and ③.

Since ①is a beam with 8' 0" depth, $\frac{H}{L} = \frac{8}{24} = 0.33$, it can be assumed that its behavior is predominantly in shear and will offer fixity to portions ②and ③at top.

The deflection of the wall is calculated below. (The entire procedure is given in *1997 Design of Reinforced Masonry Structures*, published by CMACN and is not repeated here.)

$$\Delta_2 = \Delta_3 = \frac{1}{E_m t}\left[\left(\frac{H}{L}\right)^3 + 3\left(\frac{H}{L}\right)\right]$$

$$= \frac{1}{1.5(7.625)}\left[\left(\frac{8}{10}\right)^3 + 3\left(\frac{8}{10}\right)\right]$$

$$= \frac{1}{11.44}[0.51 + 2.4] = 0.25 \text{ inches}$$

Please note 82% $\left(\frac{2.4}{2.91}\right)$of deflection is due to shear.

$$\Delta_{2+3} = \frac{1}{\frac{1}{\Delta_2} + \frac{1}{\Delta_3}} = \frac{1}{\frac{1}{0.25} + \frac{1}{0.25}} = 0.125 \text{ inches}$$

$\therefore \Delta_{wall}$ with openings = Δw_o

$\Delta w_o = \Delta_{solid} - \Delta_{solid\ strip} + \Delta_{piers}$

8' 0" strip incorporating opening

$$\Delta \text{solid strip} = \frac{1}{11.44}\left[\left(\frac{8}{24}\right)^3 + 3\left(\frac{8}{24}\right)\right]$$

$$= \frac{0.037 + 1.00}{11.44} = 0.09 \text{ inches}$$

$\therefore \Delta_{wo}$ = 0.28 - 0.09 + 0.125

= 0.315 inches

Comparing the deflection of solid wall with wall with opening, the deflection is increased by 0.035 inches or 12.5%.

Now, if we consider that area of the wall is reduced by 16.7% (4 ft. out of 24 ft.) and influence of shear deformation is approximately $\left(\frac{63+82}{2}\right) = 72.5\%$

The deflection is expected to increase by 0.725 (16.7)

= 12.1% ≈ 12.5%.

Therefore, without resorting to detailed calculations, for walls with $\frac{H}{L}$ ratio of 0.67, influence of opening can be judged approximately by reduction in % of area for a preliminary estimate. The calculation is closer to the final number as $\frac{H}{L}$ decreases.

It is important that in final design, exact calculations should be made.

Case 2: Wall with $\frac{H}{L} = 1.5$.

Consider the following 8" solidly grouted wall with

$E_m = 1.5$.ksi. and $E_v = 0.6$ ksi.

Consider the same opening size as in Case 1. Since $\frac{H}{L} = 1.5$

$$\Delta_{solid} = \frac{1}{11.44}\left[4(1.5)^3 + 3(1.5)\right]$$

$$\Delta_T = \frac{13.5 + 4.5}{11.44} = \underline{\underline{1.57 \text{ inches}}}$$

$$R_s = \frac{1}{1.57} = \underline{0.636}$$

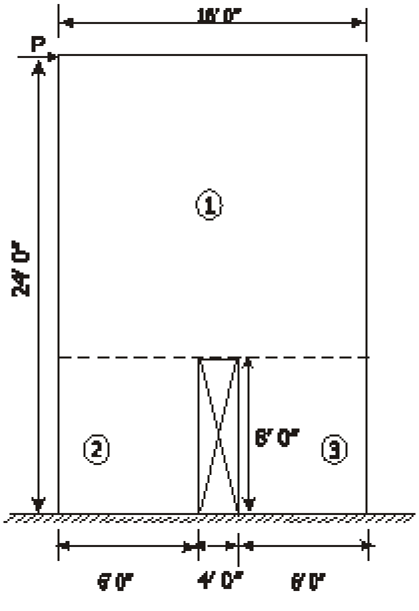

Figure 4: . Wall Elevation (Case 2)

Δ_{wo} (wall with opening)

Following the previous procedure:

$\Delta_{wo} = \Delta_T - \Delta_{solid\ strip} + \Delta_{piers}$

$\underline{\Delta_{solid\ strip}}$

$$\frac{H}{L} = \frac{8}{16} = 0.5$$

8' 0" strip incorporating opening

$$\Delta_{solid\ strip} = \frac{1}{11.44}\left[\left(\frac{8}{24}\right)^3 + 3\left(\frac{8}{24}\right)\right]$$

$$= \frac{0.037 + 1.00}{11.44} = 0.09 \text{ inches}$$

$\therefore \Delta_{wo} = 0.28 - 0.09 + 0.125$

$= 0.315$ inches

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Consider the same opening size as in Case 1. Since $\frac{H}{L} = 1.5$

$$\Delta_{solid} = \frac{1}{11.44}\left[4(1.5)^3 + 3(1.5)\right]$$

$$\Delta_T = \frac{13.5 + 4.5}{11.44} = \underline{\underline{1.57 \text{ inches}}}$$

$$R_s = \frac{1}{1.57} = \underline{0.636}$$

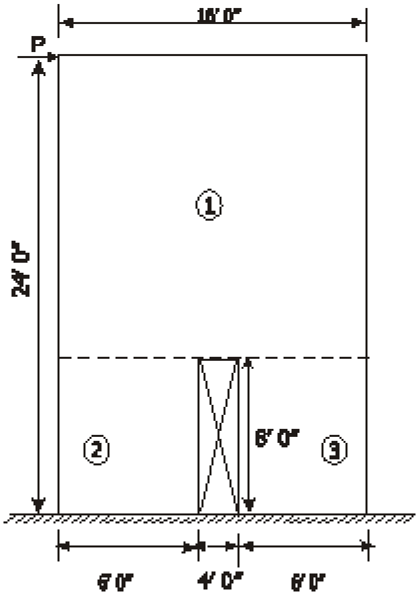

Figure 4: Wall Elevation (Case 2)

Δ_{wo} (wall with opening)

Following the previous procedure:

$\Delta_{wo} = \Delta_T - \Delta_{solid\ strip} + \Delta_{piers}$

$\underline{\Delta_{solid\ strip}}$

$$\frac{H}{L} = \frac{8}{16} = 0.5$$

$$\Delta_{solid\ strip} = \frac{1}{11.44}\left[(0.5)^3 + 3\,(0.5)\right] = 0.14 \text{ inches}$$

$\underline{\Delta_{piers}}$

$$\frac{H}{L} = \frac{8}{6} = 1.33$$

$$\Delta_2 = \Delta_3 = \frac{1}{11.44}\left[(1.33)^3 + 3\,(1.33)\right] = 0.555 \text{ inches}$$

$$\therefore \Delta_{piers} = \frac{1}{\frac{1}{\Delta_2} + \frac{1}{\Delta_3}} = 0.28 \text{ inches}$$

$$\therefore \Delta_{wo} = 1.57 - 0.14 + 0.28 = \underline{\underline{1.71}} \text{ inches}$$

$$R_{wo} = \underline{0.585}$$

The increase in deflection due to opening is 0.14 inches (approximately 9%).

The decrease in area at base is 25%. However, this is a flexure governing wall. 75% of deflection is due to flexure. So approximation based upon reduction in area is not correct.

However, since flexural deflections are governed by "I," a quick calculation of reduced moment of inertia will yield better approximation for preliminary purposes.

For a unit thickness,

$$I_{wo} = 2 \times \frac{6^3}{12} + 2(6)\,(5)^2 = 336 \text{ ft.}^4$$

$$I_{solid\ wall} = \frac{16^3}{12} = \underline{\underline{341}} \text{ ft.}^4$$

Reduction in moment of inertia is only 1.5%.

However, since 75% is flexural contribution, expected increase in deflection = 0.75 x 0.015 = 0.0112, i.e., 1.12%.

25% deflection is due to shear contribution.

Expected increase in deflection based on 25% reduction in area = 25% x 0.25 = 6.25%

Total expected increase in deflection

= 6.25 + 1.12 = 7.37 % □ 9%.

Conclusion

This discussion on deflection, and thus the rigidity of wall with openings, provides an insight on influence of flexural and shear deformations. The approximate calculations are good enough for preliminary design considerations.

One should keep in mind $\frac{H}{L}$ ratio before the approximations are used. Generally, the cutoff ratio between shear dominant behavior and flexural dominant behavior is $\frac{H}{L}$ = 1.0.

“Masonry Chronicles” is a publication of the Concrete Masonry Association of California and Nevada.

Please contact the Association Executive Director, Dr. Vilas Mujumdar, with any comments or suggestions for future issues.

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